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On the use of chaotic dynamics for mobile network design and analysis: towards a trace data generator

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Abstract

With the constant increase of the number of autonomous vehicles and connected objects, tools to understand and reproduce their mobility models are required. We focus on chaotic dynamics and review their applications in the design of mobility models. We also provide a review of the nonlinear tools used to characterize mobility models, as it can be found in the literature. Finally, we propose a method to generate traces for a given scenario involving moving people, using tools from the nonlinear analysis domain usually dedicated to topological analysis of chaotic attractors.

Keywords Mobile networks; Mobility models; Chaotic dynamics; Poincaré section; First return map

1 Introduction

The number of applications that use autonomous devices, for instance robots or Unmanned Aerial Vehicles (UAVs), increases nowadays. In this context, defining and analysing their mobility is particularly important. A mobility model describes the behaviour of an entity considering its capacities, possible moves and speed. The mobility models are described either analytically at the individual level, or by the interactions between the parts of the system (between UAVs, UAVs and planes, UAVs and points to survey, etc.). The resulting behaviours described with these simple rules can induce the emergence of a global intelligent behaviour. Inversely, from the resulting behaviour of such a swarm, these initial simple rules are hard to discover. A similar phenomenon

occurs for chaotic dynamics where a chaotic process appears to be random while it arises from a deterministic process. Therefore, the idea is to study the connections between the two concepts.

Literature on *chaotic dynamics* and *nonlinear dynamics* has been developed at the end of the century while the main concepts of this theory come from the early nineties (see Fig. 1 of [1]). Chaos is observed, described and analysed in numerous domains, for instance: electronic circuits [29], chemical reactions [47], laser behaviours [30] or biological models [62]. The first example of the use of chaos (from the Chua system [9]) to design a mobility model has been proposed in 1997 [49]. The authors proposed to use the properties of chaotic dynamics to ensure a good coverage of an area. Chaotic dynamics is defined as follows: the solution of a deterministic process is chaotic if it is sensitive to initial conditions, aperiodic and globally time invariant. These properties induce that this chaotic solution will be unpredictable when considering a long term behaviour. Nowadays, several tools improve the understanding of a chaotic behaviour, chaotic mechanism and bifurcation diagrams and some authors use them to provide mobility models. Because “*little is known of the potential relationship between swarm and chaotic systems*” [21], the main goal of this paper is to explore the uses of chaotic dynamics in the domain of mobility models. This can be done either by designing chaotic deterministic models or by using tools coming from nonlinear analysis.

In this paper we first present a review of mobile networks including chaotic dynamics using chaotic maps or ordinary differential equations. We then review the tools used to analyse these systems. In section 3, we propose a method to generate traces using nonlinear analysis tools. This method is supported by numerical simulations using the Lorenz system and permits to reproduce congestion and distribution patterns. We finally present the conclusion along with our future work in section 4.

2 Mobility models and chaotic dynamics

In this section, we present models from the literature including chaotic dynamics and the tools used to analyse them. In these models, chaos is obtained from well-known discrete or continuous systems. The chaotic variables of these systems are used to design mobility models. Sections 2.1 and 2.2 detail the two main approaches used to generate chaotic dynamics for mobility models using: a chaotic attractor from a discrete system or a chaotic attractor solution of a continuous system. Section 2.3 is dedicated to the tools from nonlinear analysis and their applications. The last section (Sec. 2.4) details our previous contributions in this domain in which we proposed to use periodic orbits of chaotic dynamics to define mobility models of UAVs in order to enhance the coverage of an area.

2.1 Models using chaotic maps

Chaotic maps are nonlinear recurrent relations. The most used in the literature is the *logistic map* defined by $x_{n+1} = \alpha x_n(1 - x_n)$. Introduced by Verhulst [58] it represents the growth of population x at each step where α is the growth rate. This map contains only one nonlinear term and can exhibit both periodic and chaotic dynamic. When varying α from 2 to 2.4, its bifurcation diagram results in a period doubling cascade: a classical route to chaos found in several systems. For details on the logistic map, the reader is referred to [5]. Charrier *et al.* [7] propose to use the logistic map to reproduce the behaviour of flocks. In the original paper [39], Reynolds introduced *boids* to reproduce the flocking behaviour with three rules (collision avoidance, velocity matching and flock centring) that generates a force vector for each agent in the swarm. In the Charrier *et al.* model the synchronization between agents is performed by the environment: the control parameter of the logistic map is updated depending on the neighbourhood of each agent. This model does not use the standard rules of flocking but reproduces their behaviour. The authors use a *bifurcation diagram* to emphasize the convergence of the system and to show the global dynamics of their agents and the transition to chaotic dynamics.

A chaotic map (the standard map [24]) can also be used to produce chaotic motion for a robot [28]. From such a two-dimensional map, a planning is assigned to the robot using two coordinate points obtained from the chaotic map. Then a robot visits these points in their order of appearance. As the purpose of the robot is to cover a square surface, the authors evaluate their system using coverage rate. Curia *et al.* [11] proposed to use another map, the Hénon map, to move a mobile robot with unpredictable trajectories. Their mobility model combines a guiding line that the robot follows with a chaotic motion obtained from the map. While the robot follows the guideline, the chaotic motion controls the evolution of the robot around this line. Here again, the authors use a bifurcation diagram to underline the chaotic properties of their system. Even if their system is made of six equations (including the Hénon map), they prove that it has a chaotic solution when $a = 1.4$ as it is the case for the Hénon map. The additional equations dedicated to the movement around the line do not influence the chaotic dynamics.

2.2 Models using ordinary differential equations systems

In this section, we present models that use a set of ordinary differential equations as a source of chaotic dynamics. We are now considering chaotic continuous solutions instead of discrete solution from a map. However, there is a way to discretely represent these continuous chaotic dynamics. These solutions are embedded in the *phase space*, for instance the Lorenz attractor [26] is a famous attractor in a three-dimensional phase space. Introduced by H. Poincaré [37]; the *Poincaré section* is a transversal surface of the flow that provides a discrete description of the chaotic dynamics obtained from continuous systems. The original idea is to consider only the discrete points when the flow crosses

a Poincaré section instead of the whole trajectory in the phase space. The discrete sequence of points contains a synthesis of the dynamical properties of the attractors. Most of the references presented below use the chaotic dynamics from the chaotic attractor using a Poincaré section.

There are many examples of such systems where nonlinearity induces chaotic behaviour. Among the most studied one: Lorenz, Rössler, Chua systems; there are several articles about robots, the mobility models of which use a set of ordinary differential equations. For instance, Nakamura & Sekiguchi [34] use the Arnold equations to model the behaviour of a mobile robot with chaotic motion. They prove that the coverage of their chaotic robot is better than the coverage of a robot using a random walk. Further to this work, Bae *et al.* [3] proposed a “chaotic UAV” with the Chua system, Arnold equations and Van Der Pol equations. They also introduce an obstacle avoidance method without decreasing the performance of the model in terms of coverage.

Fallahi & Leung [16] proposed a cooperative set of four mobile robots synchronized using Chen [8] and Lorenz [26] systems. They used one of the variables of these systems to define the movements of their robots. In their system, one of the robots is the master, and the others are synchronized with it. This system is efficient compared to unsynchronized robots or random walks in terms of coverage rate and travelled distances. Similarly, Mukhopadhyay & Leung [33] present synchronized robots with chaotic path planners. They add a *symbolic dynamic* description of their robots. The symbolic dynamic is a nonlinear analysis tool used to describe chaotic dynamics. Its purpose is to label a discrete trajectory obtained from a Poincaré section. Thus, it gives a symbol according to the dynamical aspect of the solution depending on the topological period of the system. At the end, the solution is no longer a variable but a sequence of symbols indicating dynamical aspects of the system studied. The authors use this sequence of symbols to evaluate the synchronization rate between their robots. We also would like to mention the work of Bezzo *et al.* [4] where synchronization of chaos is used to detect changes in the topology of a mobile robotic network. The authors study the motion of mobile agents through an unknown environment with obstacles (see also [54, 55] for details on this method).

Volos and co-workers [59] proposed another use of chaos which consists in designing a path planning generator for autonomous mobile robots. From the double scroll chaotic circuit (Chua system [9]), they obtained a chaotic true random bit generator. They use it to define the path planning of the robots *i.e.*, the list and the order of the points the robots have to visit. The efficient coverage rate and the unpredictability of the robot trajectories are the main characteristics of this system. This system is similar to the system using chaotic map [28]. Comparing their coverage rate in terms of the number of planned points, the system designed by Volos and co-workers [59] is ten times better. From the same authors, similar results are obtained when the Arnold map is used to generate waypoints for path planning [12]. Finally, we would like to point out a recent work done by Pimentel-Romero and co-workers [36] using Poincaré sections of chaotic attractors as threshold to generate random numbers. They conclude that these particular Random Number Generators

(RNGs) using chaotic dynamics are efficient for generating random paths for autonomous mobile robots.

Another way to include chaos to support mobile robot mobility is presented by Rosyid *et al.* [48]. Their method does not use any well-known Ordinary Differential Equations (ODE) system to drive the robots. Robots communicate using sound that all can hear. Each robot moves in a direction depending on the “total amount” of sound received. This is modelled by an ODE that details how this system is synchronized because their relative positions influence the sound emitted and received. The authors use the coverage rate to compare their system to the previously presented methods for robots driven with Lorenz or Arnold equations to prove that their model has better performance. They also compute the *Largest Lyapunov Exponent* (LLE) to ensure that chaos occurs in their system. This value is a measure of the separation rate of two infinitely initially closed trajectories [35, 60]. The LLE refers to the predictability of a system and is commonly used as an indicator of a chaotic behaviour. This is a metric approach that does not permit to distinguish chaotic solutions that have distinct structures in the state space. The reader is referred to [6, 17, 31] for details about chaotic mechanisms (*e.g.* folding mechanism or tearing mechanism). The 0–1 test can also be used as an indicator to distinguish chaotic dynamics from periodic one (the reader is referred to [19] for details). We also mention that diffusion coefficient can be computed for first return maps to measure the difference between the dynamics when a parameter is varied [20, 23].

We recently proposed mobility models [41–43] using chaotic behaviour based on the Rössler system using Poincaré section and periodic orbits. In the next section we will present nonlinear tools used to analyse mobility models. Then, in section 2.4, we present our mobility models and the nonlinear tools used to build and analyse them.

2.3 Nonlinear analysis tools

We gave above examples of mobility models built from chaotic dynamics. Some of them have been analysed with tools coming from the domain of nonlinear analysis. In this section we present studies carried out on mobility models using these nonlinear tools to understand and describe their behaviour.

In 2014, Timme & Casadiego wrote an article entitled “*Revealing networks from dynamics*” [56]. This paper gives an overview of approaches considering collective nonlinear network dynamics, but few details are given about tools from the domain of nonlinear analysis that could be used when chaotic systems are identified. These tools are well introduced by Qu *et al.* [38] in their paper about emergence in swarming systems. The authors detail the emergence phenomenon and the following tools: Lyapunov Exponents, Attractor, Recurrence plot, Poincaré section. They also extract *periodic orbits* from a Poincaré section. The “periodic” term of periodic orbits refers to the state space and not to the time space (topological period). Periodic orbits are time invariant while the system evolves in a chaotic state (from initial condition, the solution evolves

and successively visits the unstable periodic orbits). From a chaotic time series and with a Poincaré section, orbits can be extracted, and this acquisition is a preliminary step of the *topological characterization* [17]. For dissipative systems, the purpose of this method is to obtain the structure of the chaotic mechanism from a topological invariant (the linking number) computed between periodic orbits (the reader is referred to [18] for details).

Hazan *et al.* [22] opt for this approach because the aim of their work is to classify the behaviour of robots using periodic orbits. The authors explain that their method is not based on a metric (for instance the LLE) because it is “highly sensitive to perturbations such as noise contamination” [22]. They use the topological characterization tool after building an embedding (they reconstruct a phase space from one variable) of the behaviour from the x -axis motion of the robot. They build a Poincaré section in order to extract periodic orbits and then describe the behaviour of the robots using the linking numbers between the orbits.

Das and co-authors [13, 14] propose to use Lyapunov exponents to distinguish transitions in a multi-agent swarm system. The system is designed to solve an optimization problem where the agents have to reach a particular point. Computing Lyapunov exponents enables the authors to find the range of parameters where chaos occurs and where the system is no longer periodic. The authors propose an application of their system: each robot is an automatic fire extinguisher, and they have to reach a burning place. “*In any swarming dynamics, emergence of chaos is a very important situation to be dealt with*” [13] and this is illustrated by the work of Wu and co-workers [61]. In the latter article, the authors use Lyapunov exponents to analyse their swarming system and concluded that the chaos in swarm model becomes weaker while the emergence becomes stronger. The Lyapunov exponents are also used to analyse a swarm model of Self Propelled Particles (SPP) by Shiraishi and Aizawa [52, 53]. This tool permits to understand the relations between the behaviour of the system and the number of agents: “*the Lyapunov exponents reflect the biological sensitivity hidden behind the motion of swarm*” [52].

Chaos in neuronal network is also studied with Lyapunov exponents [15] or more recently, using reconstructed attractors with time-delay coordinates and with a Poincaré section [25]. These latest tools are robust to well define the chaotic mechanism because the first return map to the Poincaré section with unimodal structure indicates that there is a stretching and folding mechanism: it is a signature of the classical “horseshoe” mechanism (also known as *folding mechanism*). This type of chaotic mechanism is also present in the work of Sato and co-workers [50, 51] as illustrated by Fig. 6 of [50]. They work on a multiagent system using reinforcement learning that is modelled with coupled differential equations. This is applied to game theory: Matching Pennies and Rock-Scissors-Paper games. The stretching and folding mechanisms describe the effect of mutual adaptation and memory loss with non-transitive structure for their system. This leads to Hamiltonian chaos if there is no memory loss and to a dissipative system where there is memory loss. The dissipative system exhibits limit cycles, intermittency and deterministic chaos. To study their

system, they employ Lyapunov exponents, Poincaré section, bifurcation diagram and extract periodic orbits. These tools are also used to study languages and learning mechanisms where chaotic dynamics appears [32].

2.4 Mobility models based on periodic orbits

We recently proposed mobility models using chaotic behaviour based on the Rössler system [41–43]. These mobility models permit to enhance the coverage of an area compared to random mobility models. We used the *first return map* from a Poincaré section of a chaotic attractor solution of the Rössler system and considered the periodic orbits to build efficient mobility models in terms of coverage rate. The Rössler system [47] is given by the equations

$$\begin{cases} \dot{x} &= -y - z \\ \dot{y} &= x + ay \\ \dot{z} &= b + z(x - c) \end{cases} \quad (1)$$

and its Poincaré section is defined as follows:

$$P = \{(y_n, z_n) | x_n = 0, \dot{x}_n > 0\} \quad (2)$$

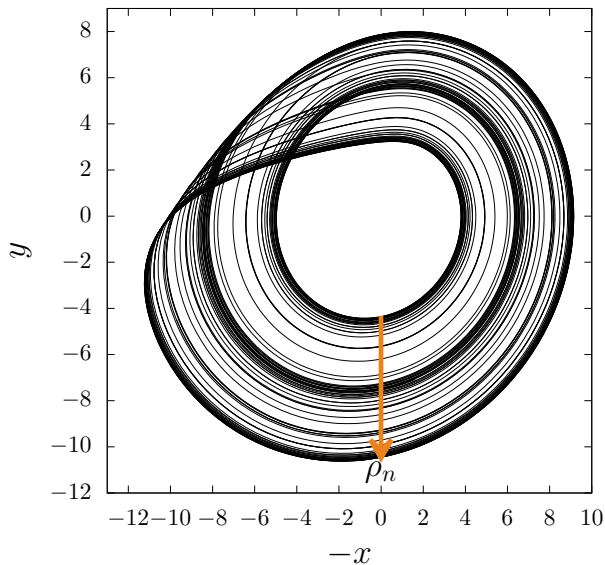


Figure 1: Chaotic attractor solution of the Rössler system (1) (values of parameters $a = 0.1775$, $b = 0.215$ and $c = 5.995$) with the Poincaré section (2) represented by an arrow.

These tools are used to obtain the topological structure of the Rössler attractor [45] where $\rho_n \in [0; 1]$ is the normalized value of y_n in the Poincaré section

(Fig. 2). We introduce a new concept to provide trajectories for UAVs: the dynamics of the first return map enable us to obtain a local direction. The first return map is a step-by-step process used to update the direction of the UAVs based on a three symbols dynamic (L for left, R for right and A for ahead). The periodic orbits of an attractor are considered as its skeleton because they structure the dynamics of the system. From the first return map, we extract the periodic orbits of attractors to obtain these recurrent points often visited with the same order. Thus, our UAVs can follow these specific patterns that allow them to explore a wide area. We obtained straight lines and wide turns with respectively period one orbit (AAAA ... for ahead, ahead, ahead, ...) and period two orbit (ARARAR ... for ahead, right, ahead, ...) (Fig. 2). The period one orbit leads to a straight forward line to enable exploration while the period two orbit enables the UAV to make large right turns to change its direction. The reader is referred to [43] for details about the periodic orbits of attractors for mobility models of UAVs. The increase in performance provided by these mobility models using first return maps indicates that they deserve further investigation. In the next section, we propose a method to generate traces for mobile agents using this specific tool from nonlinear analysis. We would like to answer the following question: from a global point of view, can first return maps be useful to produce and/or analyse traces of UAVs?

3 Traces generation from multi-components Poincaré section

In this section we first present the concept of *partial first return map* as a tool to describe data with an unknown part. Conversely to the classical first return map, this partial first return map allows input and output which is a mandatory property to provide data traces. The main purpose of this article is to analyse and generate traces of agents in an open environment which means that the agents can be added or removed in the model. Then, section 3.2 provides a scenario where a partial first return map can describe the behaviour of agents. In section 3.3 we present a proof of concept supported by numerical experiments using a theoretical dynamical system: the Lorenz system [26].

3.1 Concept

The purpose of this section is to propose a methodology to generate traces of mobile entities named *agents* (robots, persons, UAVs, ...) using the most accurate tools related to the chaotic behaviour. We consider that the agents move in a well-defined area (the environment). The agents can enter, move and exit this area after a while. We suppose that the behaviour of the agents is mainly induced by constraints of the environment that force them to follow certain paths. As a consequence, we can make an analogy between the environment and the phase space of a deterministic dynamical system. We do consider that

these paths are similar to unstable periodic orbits of an attractor in a phase space that agents might follow.

The classical method applied in this case is to reconstruct the whole phase space from one measured variable (for instance one coordinate of the position). Contrary to Hazan *et al.* [22], we do not consider that the traces contain the whole dynamical properties, but only a portion of it because of the capability of the considered robot to enter and exit the area. They use the standard way to reconstruct the phase space. For the same reason, we are not able to use even better global modelling methodology for time series data [27].

Based on our experiment in mobility models design [41–43], we can say that an approach using the orbits of an attractor as guidelines is very efficient in terms of coverage of an area. This efficiency is due to the patterns followed by the UAV during the exploration process. Such an approach including patterns repetition can be applied to the generation of traces of mobile agents. Fig. 2 is the first return map of a Poincaré section of a Rössler attractor used in several research projects [41–43]. This first return map is an unimodal map made of an increasing branch and a decreasing branch: this illustrates the folding mechanism (“horseshoe” mechanism). However, this first return map details the whole chaotic dynamics of a bounded and globally time invariant process of a given chaotic attractor. Thus, the periodic points of Fig. 2 describe the entire periodic orbits and considering the analogy with the mobility model, this prevents entrance or exit of agents. As we are aiming to obtain a trace data generator, this global perspective is a drawback to overcome, because the agents cannot be considered as permanently evolving in a dedicated well-defined environment. Therefore, we propose a new method using partial data from a chaotic attractor.

For attractors bounded by high genus torus, the Poincaré section is made of several components that can be used to properly describe chaotic dynamics using symbolic dynamics [45]. For instance, the multispiral chaotic attractor (Fig. 3) introduced by Aziz-Alaoui [2] is bounded by a genus-5 torus. To describe the dynamics in a discrete way, the Poincaré section has to detail the transitions between the spirals. Consequently, the Poincaré section will be made of several *components*. These components are chosen accordingly to the bounding torus theory [57] using the fixed points. For the multispiral attractor (Fig. 3) this theory indicates that four components are required to build the Poincaré section. The Poincaré section is no longer one plane but a set of planes. The flow crosses these planes and from a continuous flow we obtain discrete values. We used a concatenation of these values to build one variable representing the whole Poincaré section: ρ_n . Each component is represented by a range of values in ρ_n : component c_i for $\rho_n \in [i - 1, i]$ (see Fig. 3). Thus, ρ_n is a variable between 0 and 4 that describes the Poincaré section. The four components synthesized in ρ_n describe the entire dynamic of the system.

In a first return map to this Poincaré section, orbits can be extracted, and a symbolic dynamic can be assigned as it has been done for the Rössler attractor (See Fig. 2 that details periodic points associated to periodic orbits). Fig. 4 shows a first return map of a Poincaré section made of four components for the

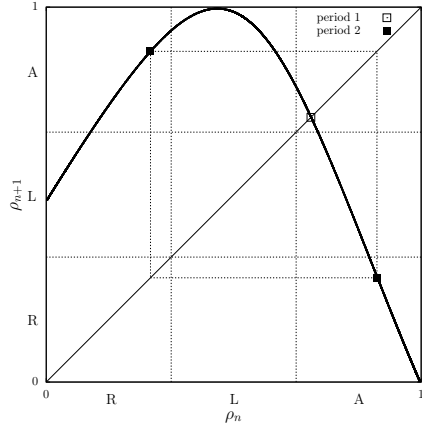


Figure 2: First return map to the Poincaré section of the Rössler attractor (Fig. 1). This map is partitioned in three parts that give the UAV directions: L (left), A (ahead) and R (right). Orbits of period 1 and 2 illustrate patterns (AAAAA...) and (ARARA...), respectively straight lines and large turns. These patterns are efficient to cover an area with UAVs [43].

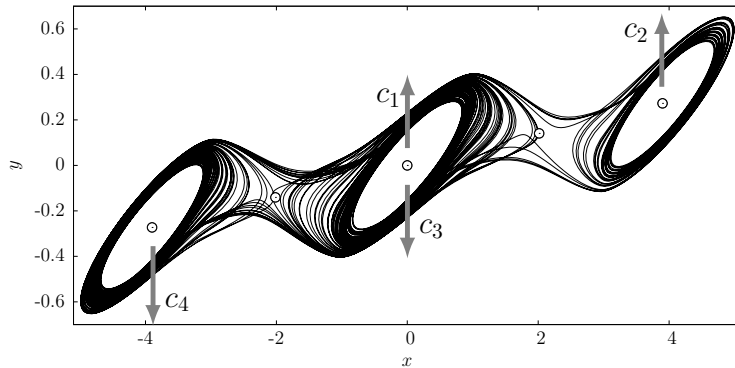


Figure 3: The multispiral attractor defined in [2] with the fixed points and the four components of the Poincaré section: c_1 , c_2 , c_3 and c_4 [45]. This attractor is bounded by a genus-5 torus (the five aligned holes of the bounding torus are the fixed points and indicated with a dot in a circle \odot).

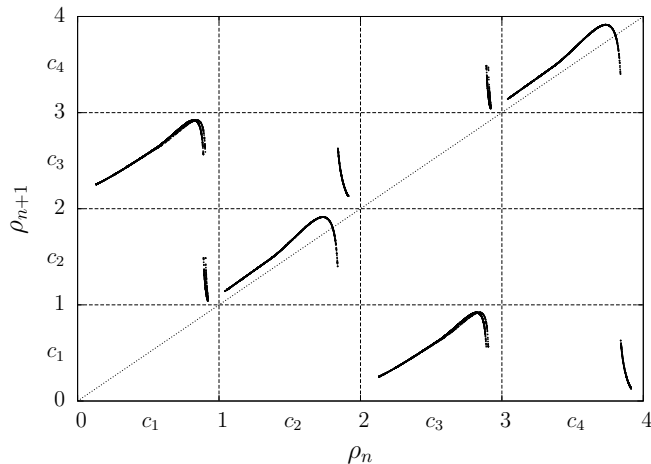


Figure 4: First return map of a Poincaré section describing a multispiral attractor (Fig. 3) using a Poincaré section made of four components [45].

multispiral attractor (Fig. 3) by plotting ρ_{n+1} versus ρ_n . This first return map contains both horseshoe mechanisms and tearing mechanisms. This map is a discrete description of the flow and permits to obtain information concerning possible transitions between components of the Poincaré section. The reader is referred to [45] for details on the methodology used to build a Poincaré section with several components using the properties of fixed points and the bounding torus theory. One of the way to build a bounding torus is to place the hole where the fixed point of the attractor are. For the multispiral attractor there are details in [45] and for the Rössler system, there are details in [40] including parameters variation ensuring the robustness of the method.

In this multispiral attractor there are transitions from a component to the same component (e.g. c_2 to c_2) and transitions to another component (e.g. c_2 to c_3). The first return map (Fig. 4) details the chaotic mechanism occurring to perform these transitions and describes the whole dynamics of the system. The novelty of our approach lies on the use of only a part of the data to introduce chaotic mechanisms as a model for traces generation. We develop a new tool to handle this unknown part of the data: the *partial first return map*. A partial return map is an incomplete map with at least one component with incoming flow, the *initial components*, and at least one component with outgoing flow, the *final components*. The flow is split in such a way that the partial first return map describes the transitions between the initial components and the final components using *transitional components*. With this repartition of components, we propose to follow a particular ordering to build the partial first return map: the initial components, the transitional components and the final components.

Our new concept is to consider only a part of the trajectory in the environment. Thus, a *partial first return map* is only an uncompleted first return map

with transitions between a subset of components of a Poincaré section. This permits to represent experimental data without taking into account the rest of the trajectories. Consequently, this missing or unknown data can be considered as input and output for traces model indicating where agents enter or leave the environment. In the next section we first present a scenario for our methodology and in section 3.3 we detail the method to build partial first return map.

3.2 Scenario

We consider the following scenario. The environment is an exhibition centre where we want to reproduce the behaviour of the visitors. We consider one entry door and one exit door. The exposition is composed of two rooms (Fig. 5). The room 1 is accessible from the entry and the room 2 is before the exit. Visitors can stay in room 1 as long as they want before leaving the exhibition. As there are two rooms to visit in the exhibition centre, we consider the transition between these rooms. Thus, we have visitors coming from the entry door and going to the first room. There is a transition from the entry to room 1 and also from room 1 to room 2.

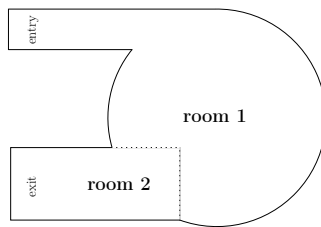


Figure 5: Scenario using an exhibition centre with two rooms. The shape is intentionally similar to the considered phase space (Fig. 6) to underline the asset of our method but this is not mandatory: only transitions between components are significant.

These transitions refer to the way chaotic multispiral attractors are analysed. As there are multiple spirals, and since the trajectory evolves from one spiral to another, it is required to consider these transitions. The Poincaré section is composed of several components to handle these multiples transitions. The analogy of transition between spirals is with transitions from one room to another room to generate traces. Moreover, this approach does not prevent to include more components to highlight patterns as it has been done to provide templates of attractors (See Fig. 15 of [45]). For instance, for a Malasoma attractor bounded by a genus one torus, a Poincaré section made of four components has been used to detail torsions or permutations movements [46]. This kind of mechanism can be used to reproduce the movements of the agents.

In this particular scenario, we consider some parts of our model as components of a Poincaré section: the entry and the exit as two components of a Poincaré section and the transition between room 1 and 2 is another compo-

ment. To build the first return map, we set up the following order: the entry is an initial component, the transition between room 1 and 2 is a transitional component and the exit is a final component.

3.3 Numerical experimentation using the Lorenz system

We perform experimentations using the Lorenz system [26]

$$\begin{cases} \dot{x} = \sigma(y - x) \\ \dot{y} = Rx - y - xz \\ \dot{z} = -\beta z + xy \end{cases} \quad (3)$$

in order to illustrate our methodology on a chaotic attractor bounded by a torus with a genus higher than one. This system is solved using a 4th order Runge-Kutta method. We obtain an attractor solution to this system for the parameters values $R = 70$, $\beta = \frac{8}{3}$ and $\sigma = 10$. Fig. 6 represents the projection of the attractor in the plane (x, y) . The hatch part is not used to perform the experimentation of our method because we do not consider the whole solution. As a consequence, a partial first return map is obtained from the data (Fig. 6) with a list of three components:

- initial component A : Entry
- transitional component B : Transition where an agent decides to stay in room 1 or to proceed to room 2
- final component C : Exit

These components are given by the following equations:

$$\begin{aligned} \mathcal{P}_A &= \{(y_n, z_n) | x_n = 0, \dot{x}_n > 0\} \\ \mathcal{P}_B &= \{(y_n, z_n) | x_n = 10, \dot{x}_n < 0\} \\ \mathcal{P}_C &= \{(y_n, z_n) | x_n = 0, \dot{x}_n < 0\} \end{aligned} \quad (4)$$

The three components are represented Fig. 6 with arrows showing the flow of the attractor between them. Here we choose arbitrarily 10 for our partition based on bounding torus theory, but we remind that fixed point of the differential equations system can also be used to ensure good partition according to this theory.

As we have done for attractors bounded by high genus torus [45], we build one variable ρ_n with different values to represent the position of the component in the partial first return map depending on its value:

- initial component A : $\rho_n < 1$;
- transitional component B : $1 < \rho_n < 2$ and
- final component C : $2 < \rho_n < 3$.

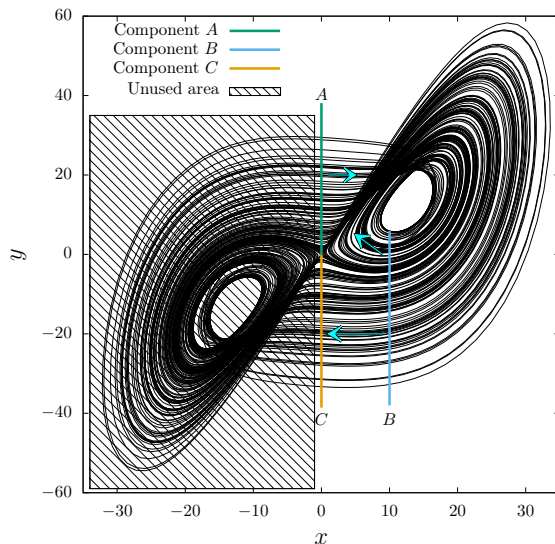


Figure 6: Phase portrait of an attractor solution to the Lorenz system (3) for the parameters values $R = 70$, $\beta = \frac{8}{3}$ and $\sigma = 10$. Arrows highlight the possible transitions from a component to another component (A to B , B to B or C).

As introduced in one of our previous work [44], we choose to follow the orientation convention that gives the values of each component from the inside to the outside of the attractor. This convention is mandatory to compare chaotic mechanisms of attractors [44]. Thus, the ρ_n values close to and lower than 1, 2 and 3 are respectively associated to positions in the components close to the letters A , B and C (Fig. 6).

We build the partial first return map (Fig. 7) based on ρ_n to highlight the possible transitions between all components. It describes the transition between components $X \rightarrow Y$ with X the abscissa and Y the ordinate:

- The absence of points with ordinate value for the component A underlines the fact that it is an initial component. In this map, it is not possible to reach component A .
- $A \rightarrow B$ in the case (A, B) and $B \rightarrow B$ in the case (B, B) indicate that the next choice for the agent depends on the value of component B : stay in room 1 or proceed to room 2.
- $B \rightarrow C$ in the case (B, C) indicates that the agent will leave the room 1 to the room 2 and then leave the exhibition
- The absence of points with abscissa value for the component C underlines the fact that it is a final component. From component C there is no successor point.

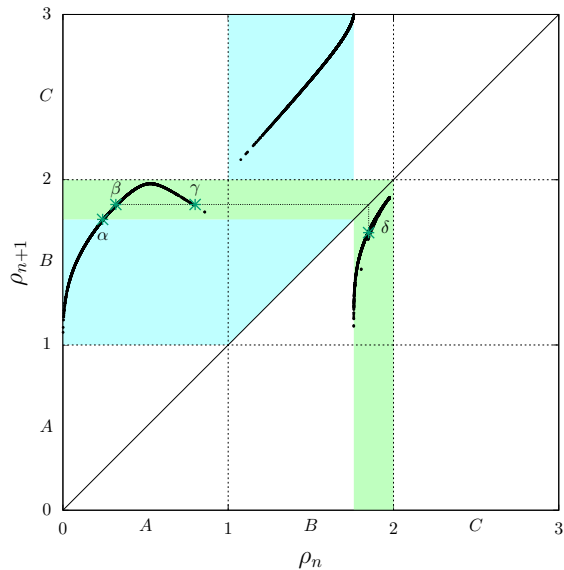


Figure 7: Partial first return map based on ρ_n illustrating the possible transitions between the components. Even if some parts of this figure are empty, we chose to show them to make the partial first return map mechanism understandable. Transitions from A to B are possible and generate congestion; for instance, β and γ have the same image (δ). Conversely, after the component B , there is a tearing mechanism that results in by the separation of closed points (the green and cyan areas underline this split).

From the dynamical point of view, even though we do not have the whole dynamical system, such kind of maps gives details on the possible chaotic mechanisms of the system. For instance, if we only consider the (A, B) area, it is an unimodal map with an increasing and a decreasing branch describing the classical “horseshoe” mechanism (stretching and folding mechanisms). This is illustrated by the positions of the points β and γ in the partial first return map (Fig. 7). These two points have the same ordinate but not the same abscissa. Using the map, we obtain only one image from these points: the point δ . In the range $[\beta; \gamma]$ there exist pairs of points in that interval for which this happens because there is only one image for two fibres. In terms of mobility model analysis, it means that even if two agents do not come from the same position in the entry (component A) they can reach the same position in the next component B . Such a mechanism illustrates congestion, *i.e.* the convergence of agents to the same point. The topological description of a chaotic attractor can be viewed as a series of mechanisms: stretching and folding are enough to generate chaos. Considering the flow of the attractor, a folding mechanism is responsible for gathering trajectories before a stretching mechanism. The congestion is the result of a folding mechanism (continuous map with several branches) because

trajectory will collapse to the same area.

Now considering points with abscissa in component B , their ordinate are in components B and C . For an agent coming from component B , there is a split into two different places. The chaotic mechanism associated to such behaviour is the *tearing mechanism*, which is well known for Lorenz attractors (for details about mechanisms in the Lorenz system, see [6]). The coloured areas (Fig. 7) underline such a split directly from component A where two points (agents) close to α will move to B but one will stay in component B (green area of Fig. 7) while the other will proceed to component C (cyan area of Fig. 7). This mechanism is similar in chaotic In the mobility model, it highlights the fact that agents can stay in the same room or proceed to the last room before leaving the exhibition.

Finally, we can use one dynamical system (*e.g.* Lorenz system) to generate the global pattern of the traces with respects to some conditions based on the transition between areas. This is a proof of concept validated by numerical simulations, a preliminary step before considering experimental data instead of a solution to a dynamical system. From data, we are expecting to find some patterns similar to tearing mechanism or folding mechanism by identify their dynamical signature in partial first return map generated from the data. Between the mechanisms of transitions in experimental real world data we will then look for a dynamical system with similar dynamical properties to use it as a trace generator. The dynamical system will not be directly extracted from the data with an algorithm. One differential equations system, with a set of parameters, has to be found in a database of research articles detailing topology of chaotic attractors (including the list of chaotic mechanisms). The additional search of an appropriate dynamical system including equations and parameters can be achieved using a method we developed providing templates of attractors directly from a bifurcation diagram [40]. This article illustrates the richness of non-equivalent chaotic dynamics that could be found in one dynamical system, and it also provides various chaotic mechanisms with their parameters. To complete, we have to mention that multiple attractors can provide same mechanisms. For instance, the multispiral attractor can be used instead of the Lorenz system because they share common chaotic mechanism. Once a system has been found, the traces can be obtained by solving the dynamical system with initial points (the mobile agent) in the initial components to let them evolve in the environment by visiting the other components and escape via the final components. We thus propose to find a model fitting the data by giving a similar partial first return map. This model could provide a good mobility model, even if there is a suppressed or hidden part that is not used. We consider agents as particles in a flow where the chaotic dynamics provide enough variability despite the deterministic process to be used as a mobility model with our methodology.

4 Conclusion

In systems containing mobile entities, we emphasize the fact that chaos can be built, as well as it can emerge, by synchronization of the entities or by interaction with the environment. In both cases, if a chaotic state is observed, then dedicated tools can be employed to analyse it. For instance, bifurcation diagrams can illustrate transition from limit cycles (periodic solution) to various type of chaotic dynamics. We have seen that, Lyapunov exponents are indicators to distinguish chaos from hyper-chaos. However, for chaotic dynamics, this measure is not well adapted to detail the chaotic mechanism. Thus, others tools (Poincaré section or periodic orbits for instance) leading to topological characterizations can separate non-equivalent chaos, and provide more accurate analysis of these chaotic dynamics. The recent improvements concerning comparison of chaotic attractors and topological characterization method can be used to identify or distinguish chaotic mechanisms and consequently identify and distinguish particular behaviours of dynamical mobile networks.

Consequently, we have focused on the dynamical structure exhibited by the transitions between components of a Poincaré section. These transitions are significant and reliable to design a mobility model with congestion or distribution of agents in a given area. The procedure is applied on a Lorenz system to highlight the advantages of our methodology. The exhibited structure can be used to both generate traces and analyse them. For instance, with the constant increase in the number of connected devices carried by users, there are several ways to collect real traces with their approval. It has been done for the students of the University Politehnica of Bucharest [10] where their social interactions have been studied. In our future work we will apply our methodology to analyse agents traces obtained by such measurements to reproduce and generate traces by finding the most appropriate dynamical system.

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